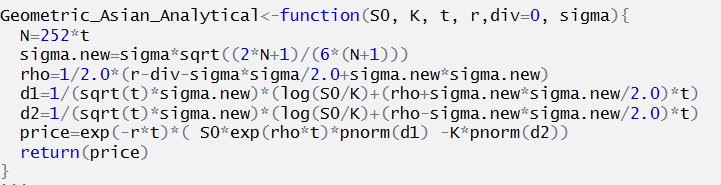
621 Final Exam

**Submitted by:**

**Saeed Rahman**

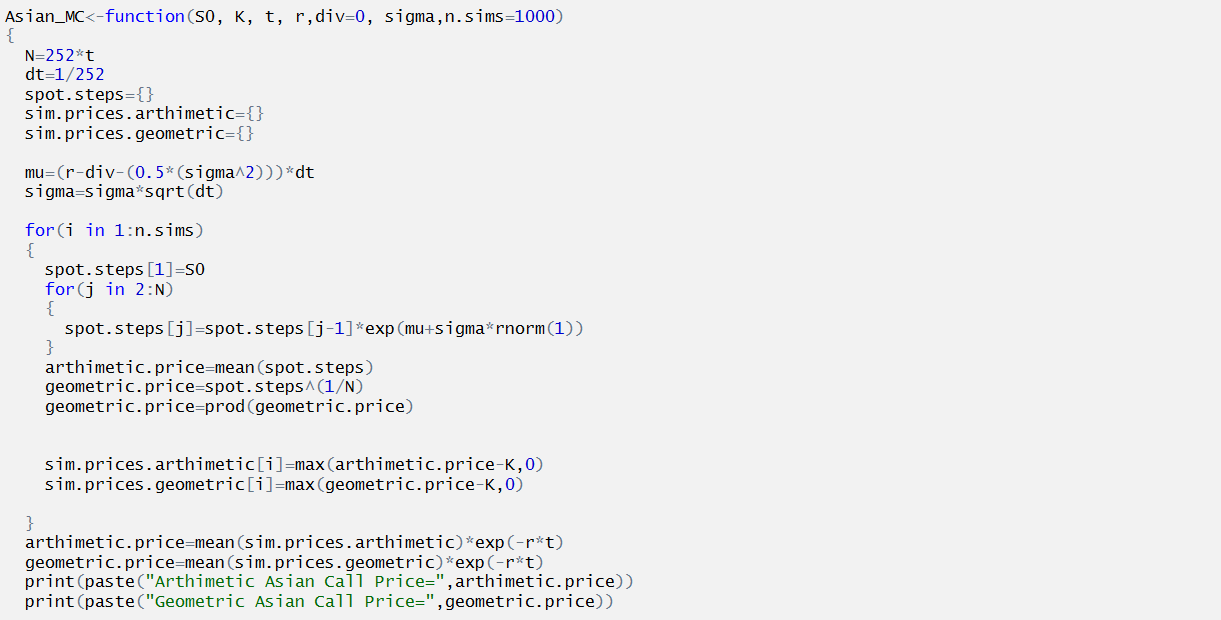
# QuestionA

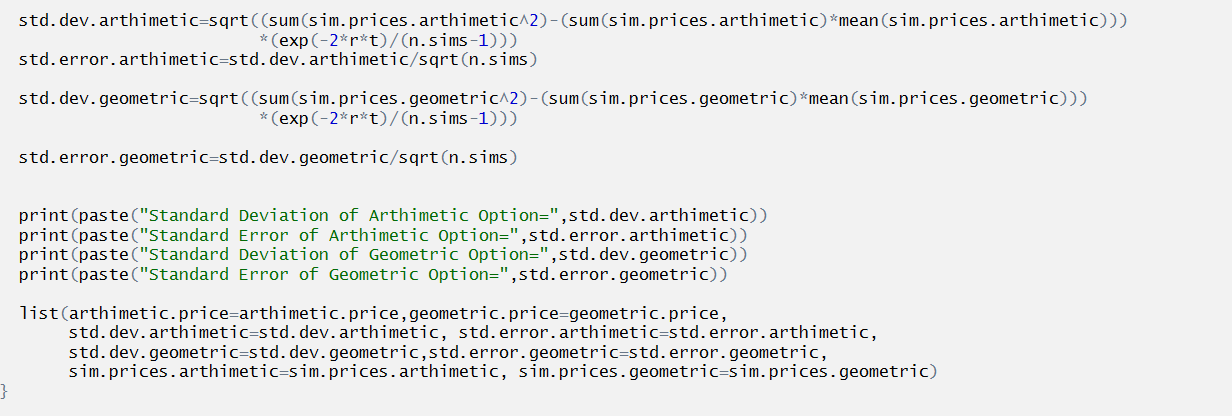
## Analytical Price of an Geometric Asian Call Option





## Monte Carlo function to price Arithmetic and Geometric Asian call option

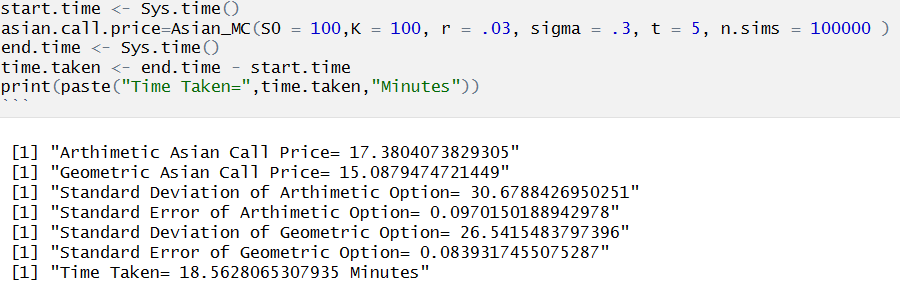




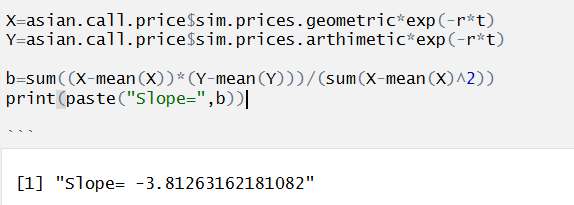


Number of simulations used: 100,000 due to time constraint

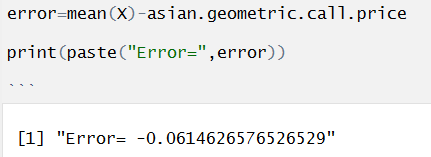
## & C ) Monte Carlo Price of Arithmetic and Geometric Asian Option



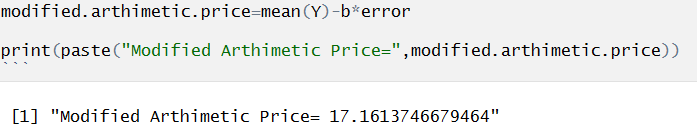
## Calculating slop/coefficient “b”



## Calculating the error



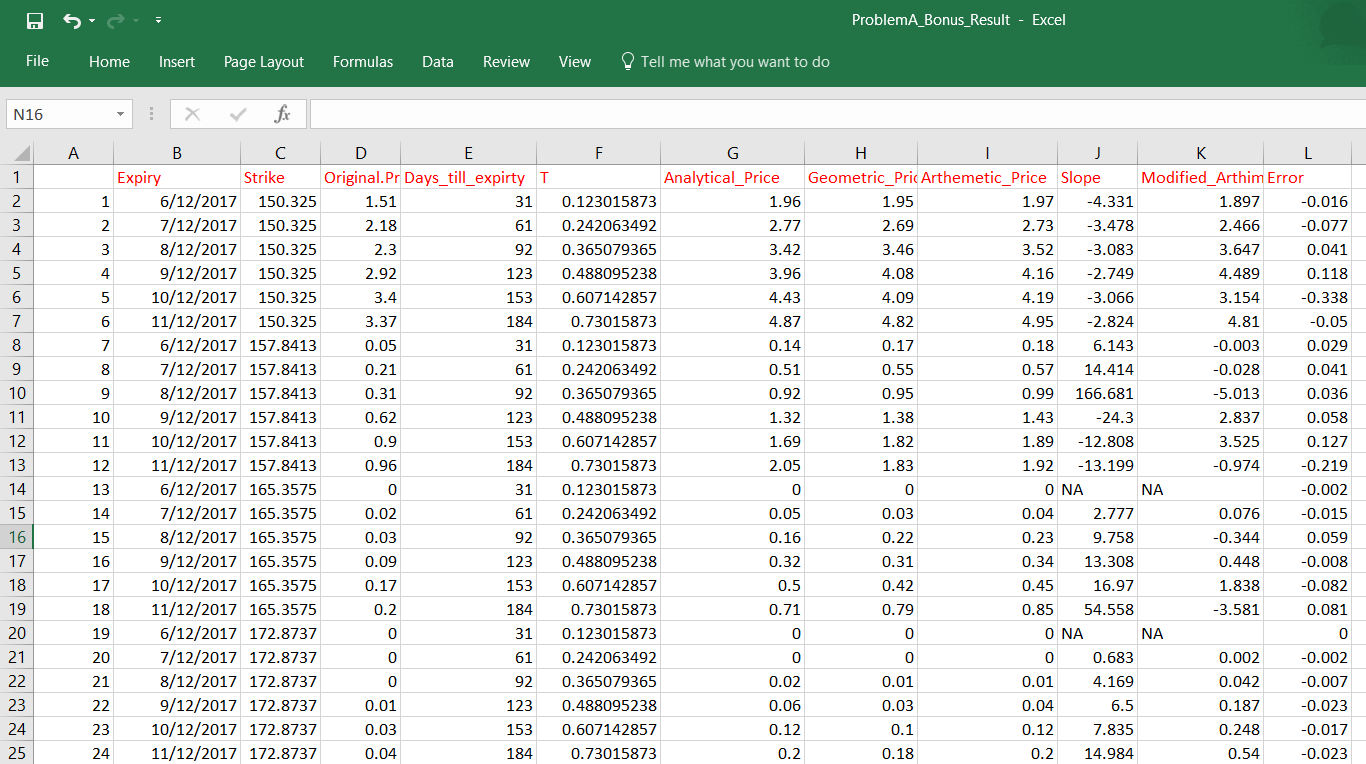
## Calculating the modified arithmetic option price



We can see that as M (number of replication) is increased the error starts to decrease and converge to a stable value.

## Applying the function to an external (IBM US EQUITY) Asian Option price calculation

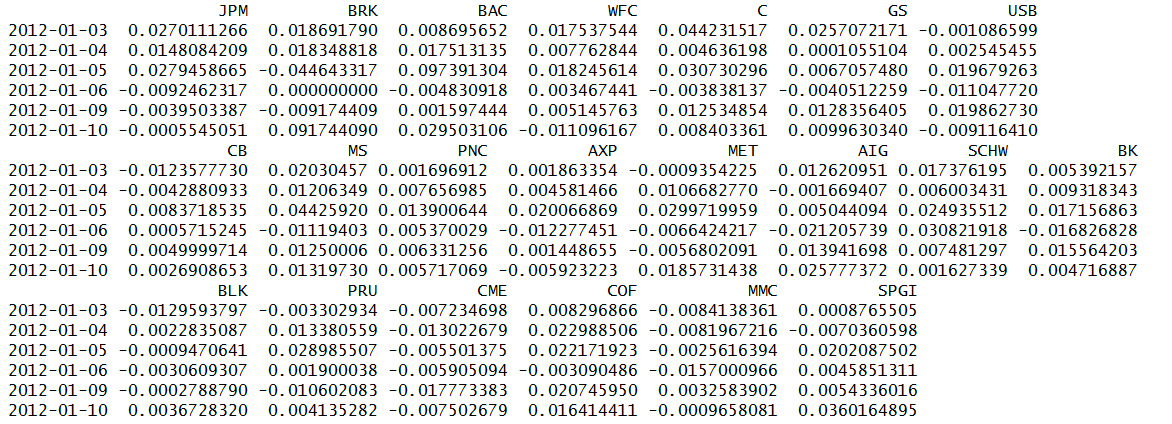




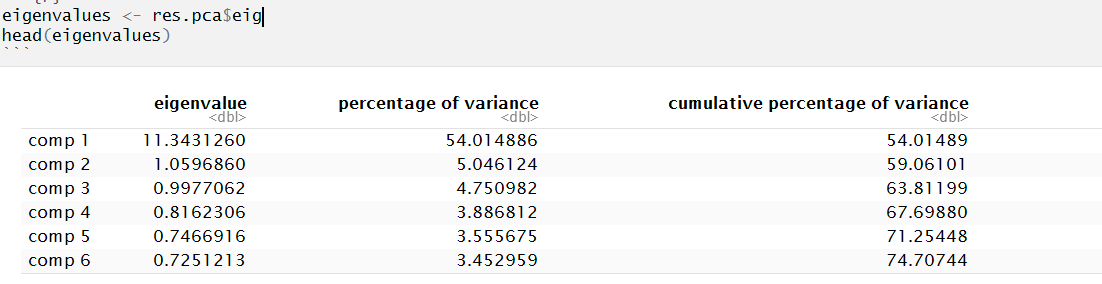
# QuestionB

## PCA

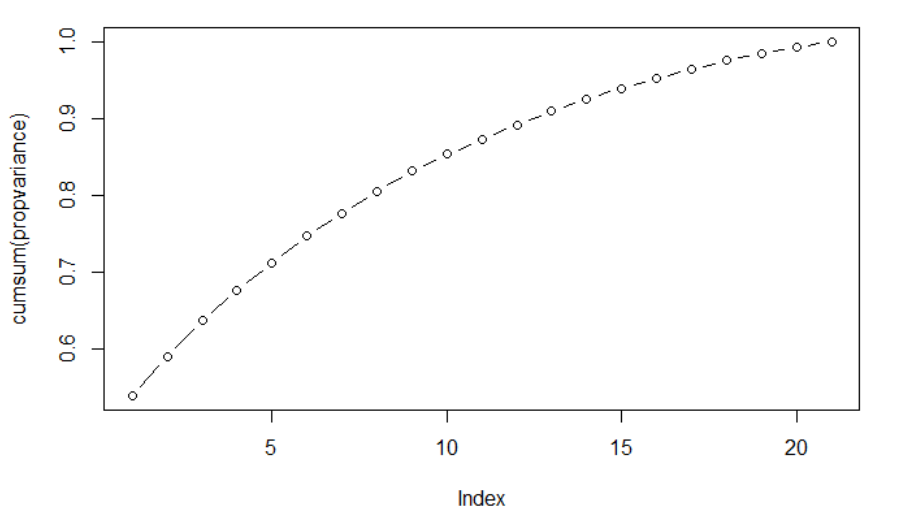
### Collecting data from 2012 for XLF ETF

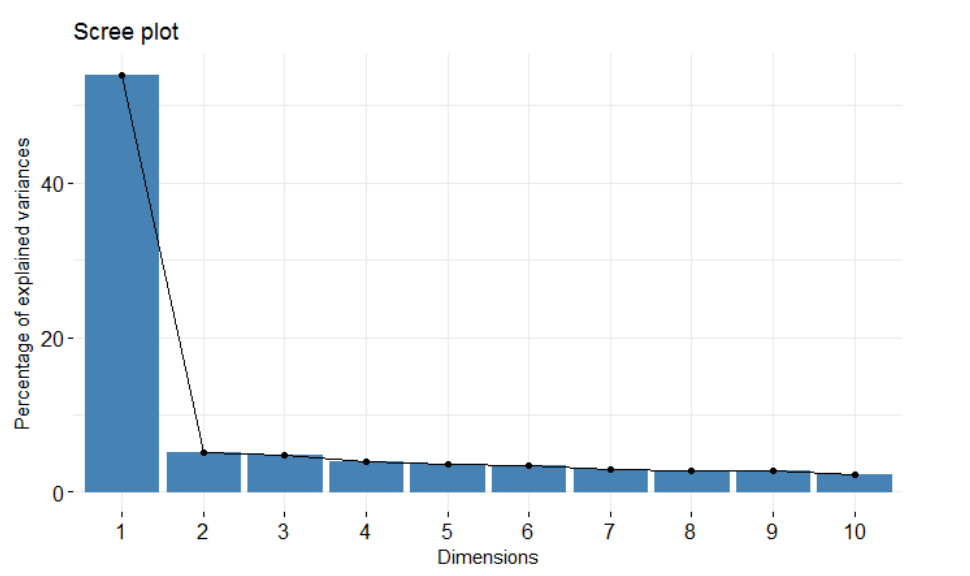


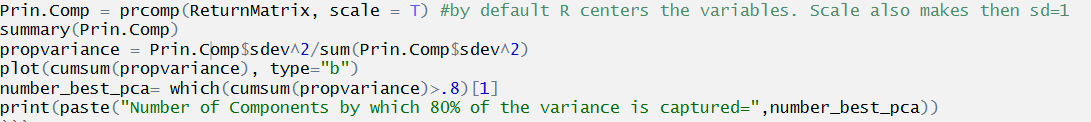
### PCA Result

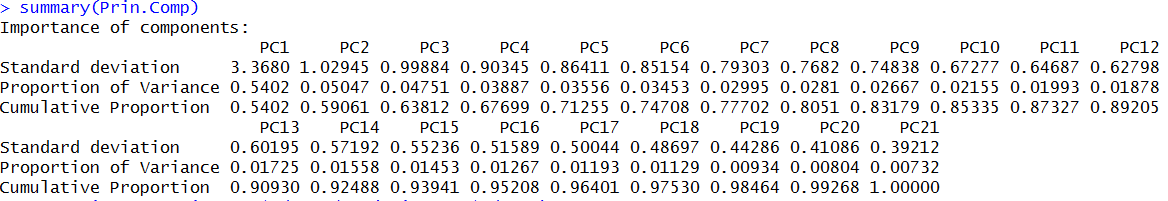


### Number of Components to account for 80% of the variability



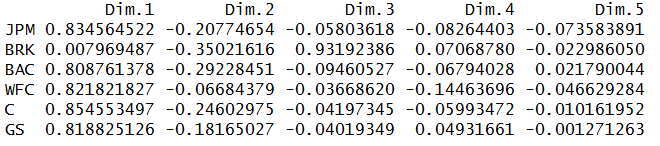




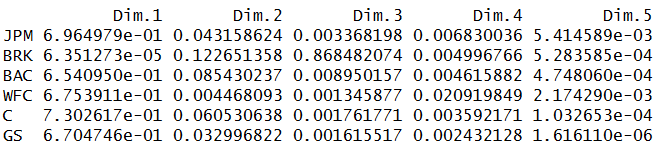


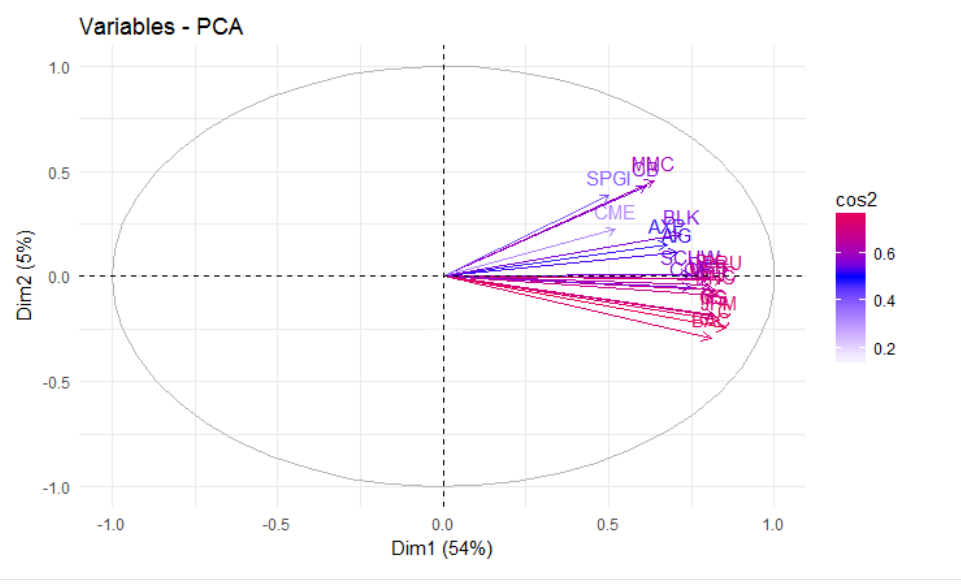


### Loadings of different Equities on PCA components



### The squared loading on the PCA components



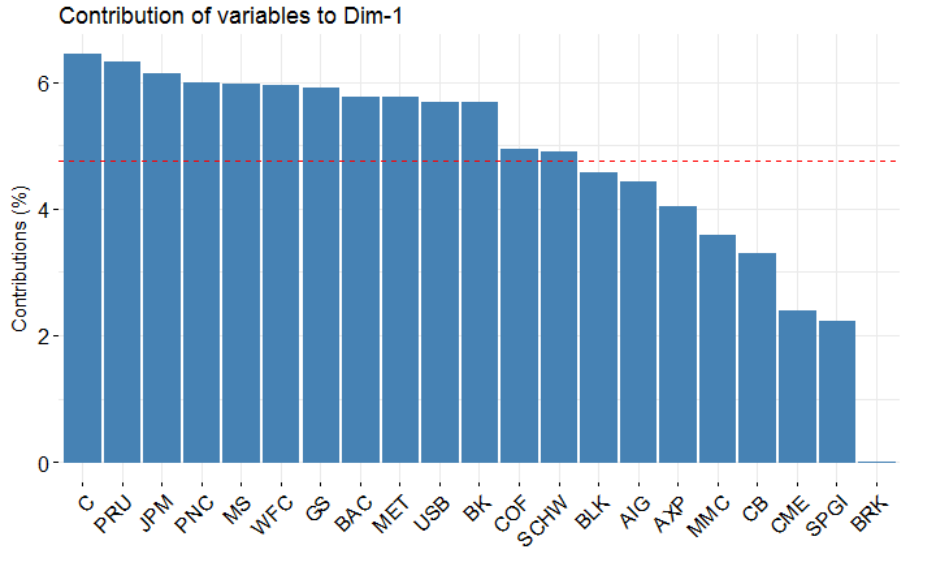


The sum of the cos2 for variables on the principal components is equal to one.

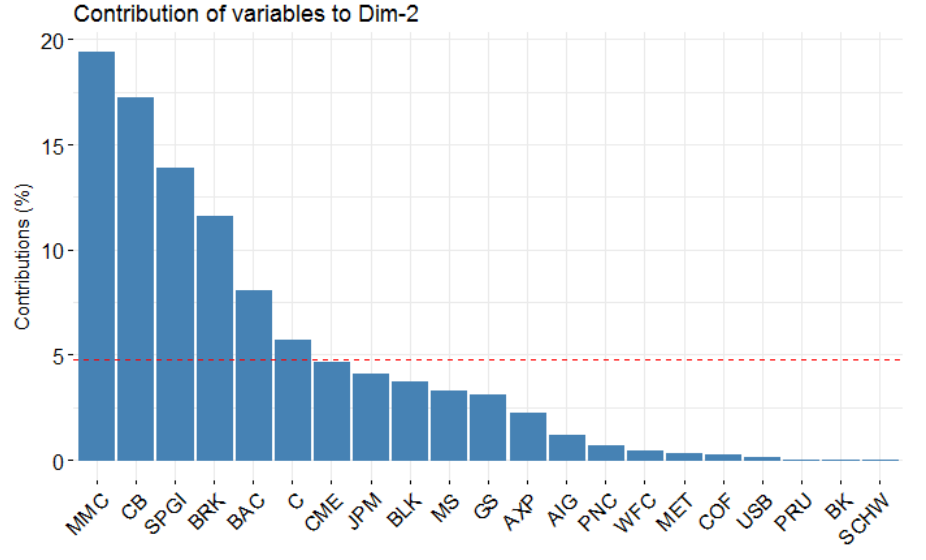
If a variable is perfectly represented by only two components, the sum of the cos2 is equal to one. In this case the variables will be positioned on the circle of correlations.

For some of the variables, more than 2 components are required to perfectly represent the data. In this case the variables are positioned inside the circle of correlations and therefore more than two components are required to represent the variance of each equities.

### Contributions of variables (equities) on PC1

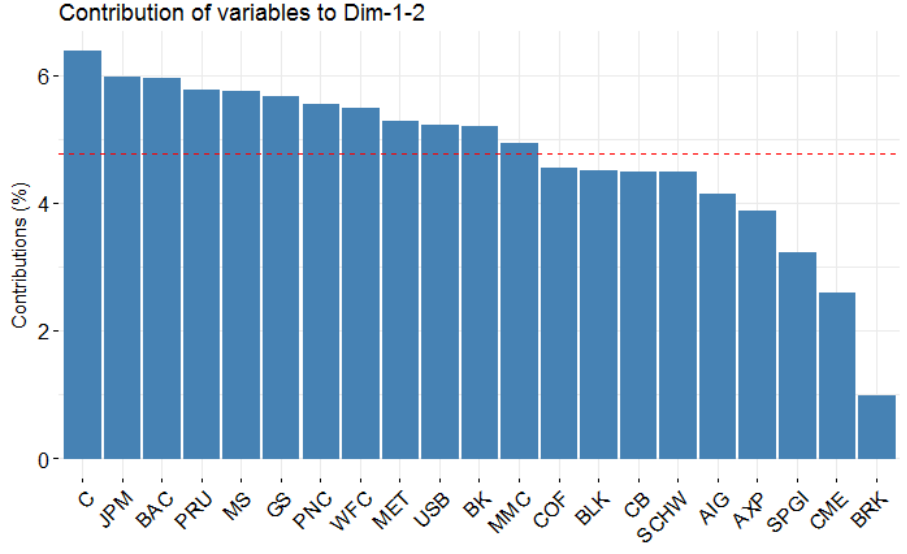


### Contributions of variables (equities) on PC2



### Contributions of variables (equities) on PC1 and PC2





## Selecting 4 top equities and fitting SDE’s to find the right model

Model 1

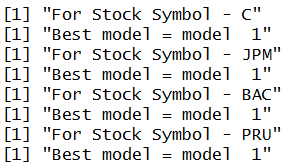
Model 2

Model 3

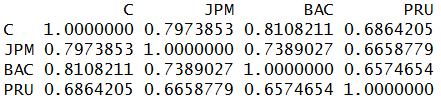
Model 4

Model 5

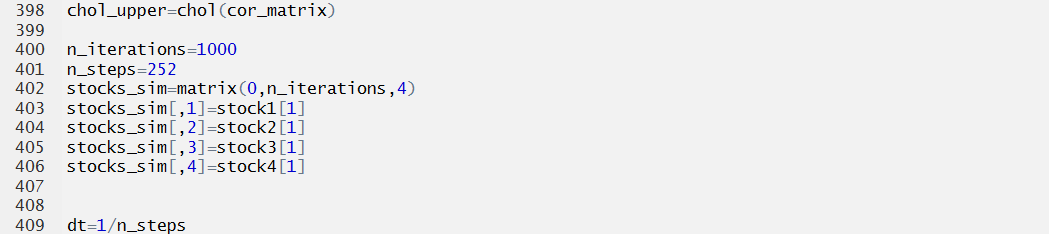
We will select the symbols **"C","JPM","BAC","PRU"**

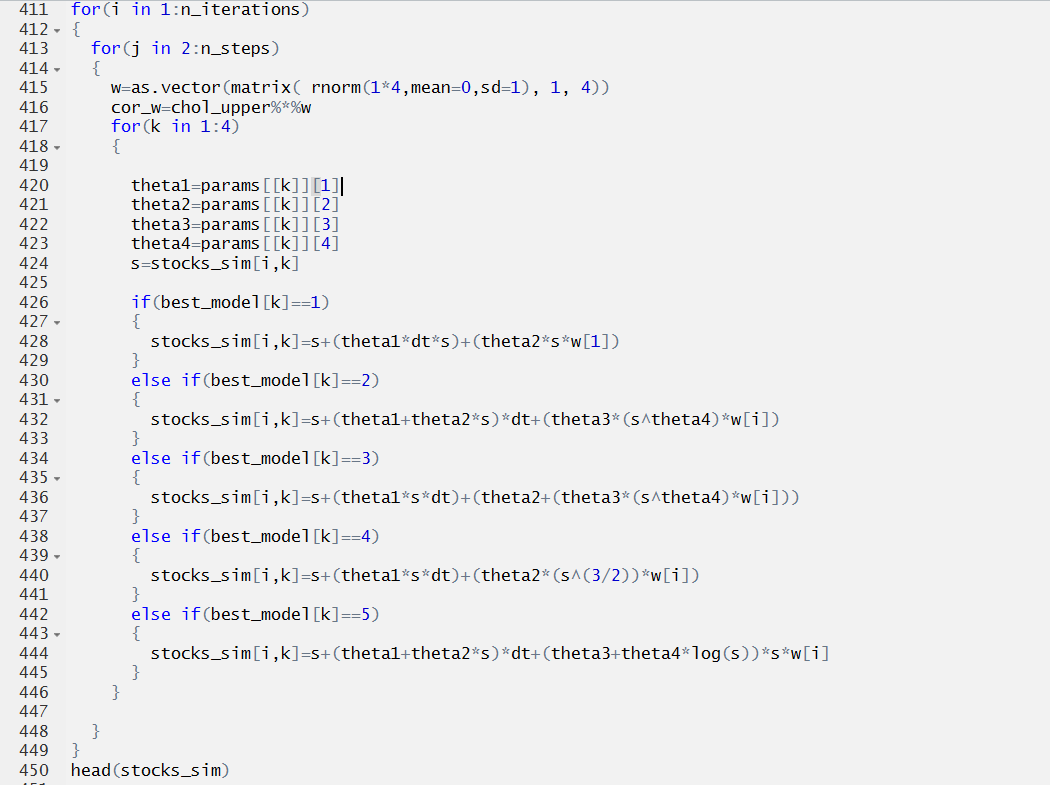


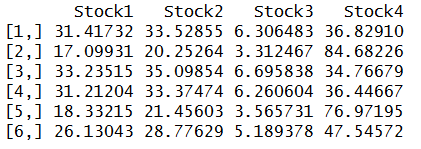
## Correlation Matrix of top 4 stocks



## Monte Carlo for the 4 stocks

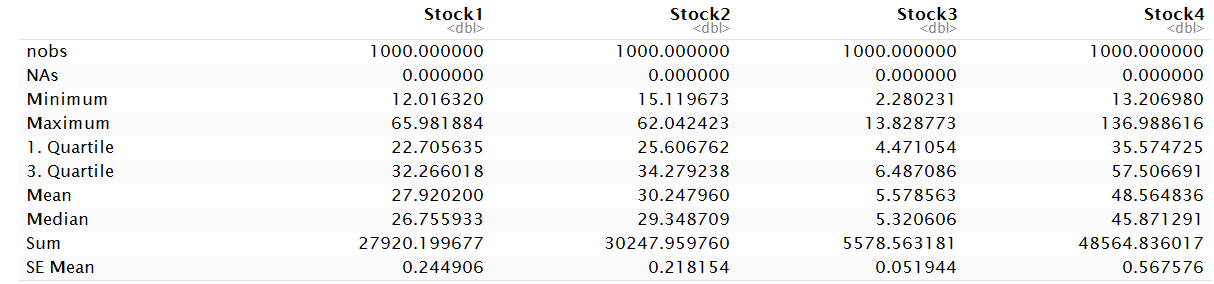


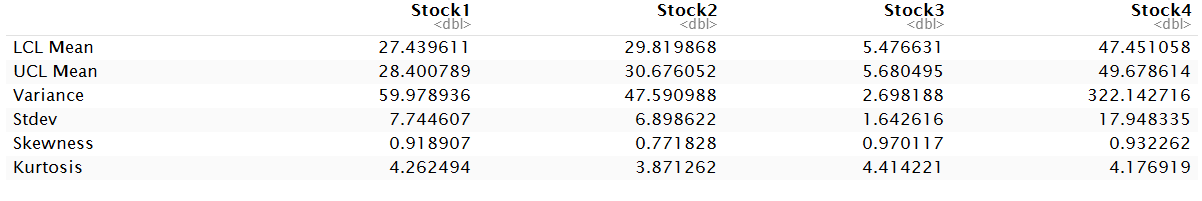




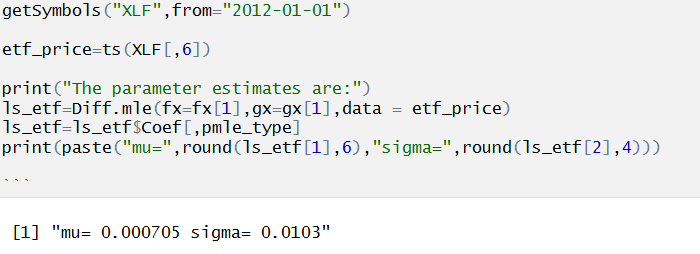
Index represents each simulation

### Basic Statistics of the simulations

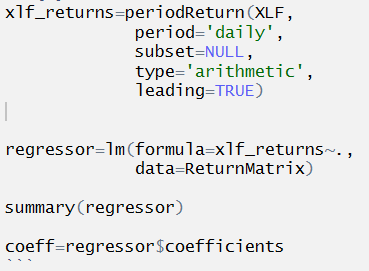


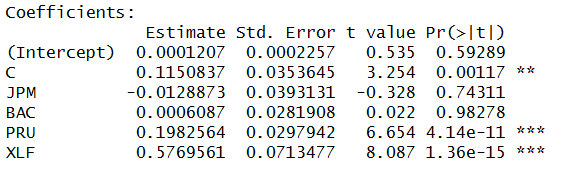


## Fitting the ETF (XLF) data

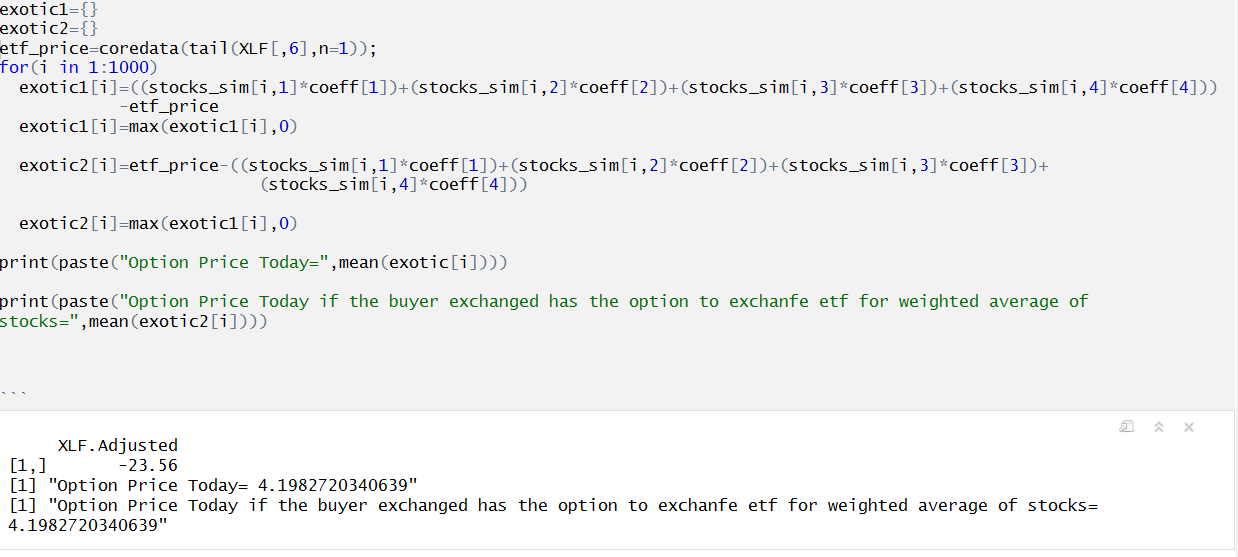


## Multivariate Regression on the 4 stocks and the ETF





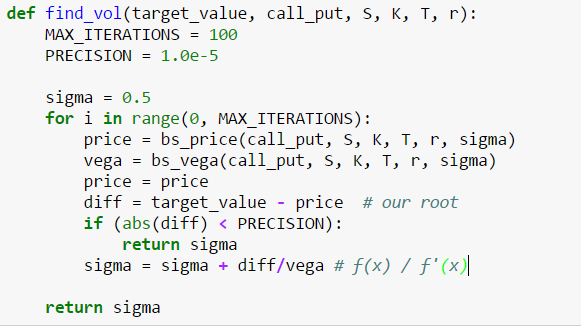
## Basket Security

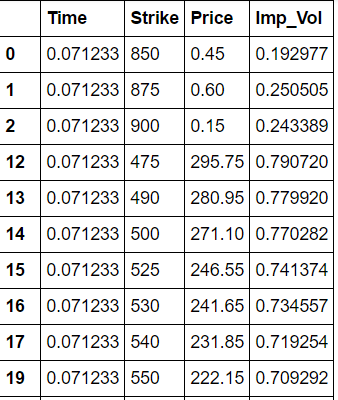


# QuestionC

## Computing the Implied Volatility

Used Newton-Raphson Method

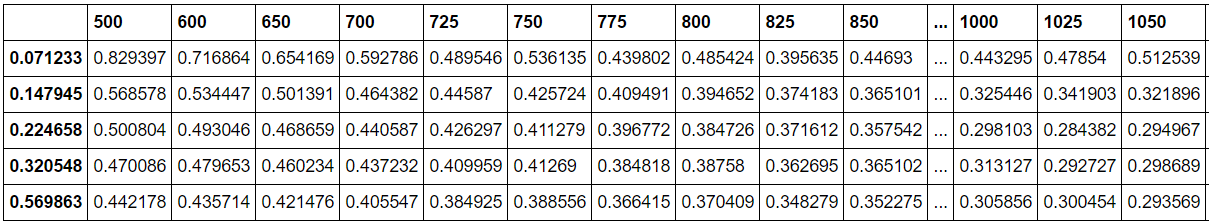


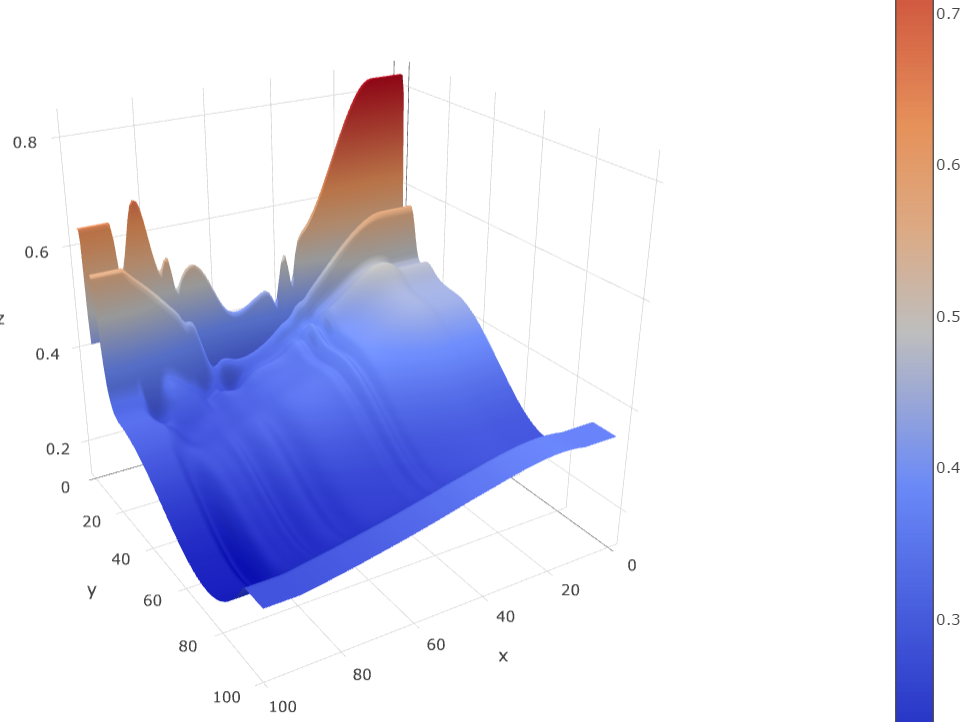


## Interpolating the Implied volatility Surface

Package Used: Scipy: Interpolate - interpolate.interp2d

Method: Cubic Spline

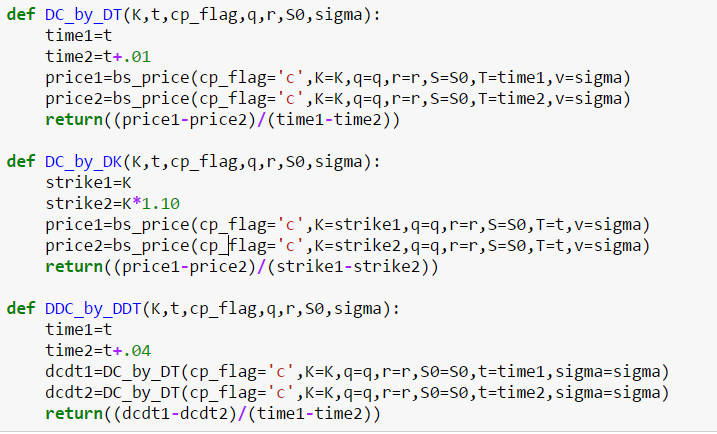


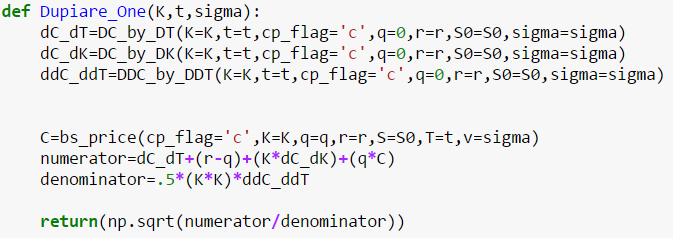


## C)

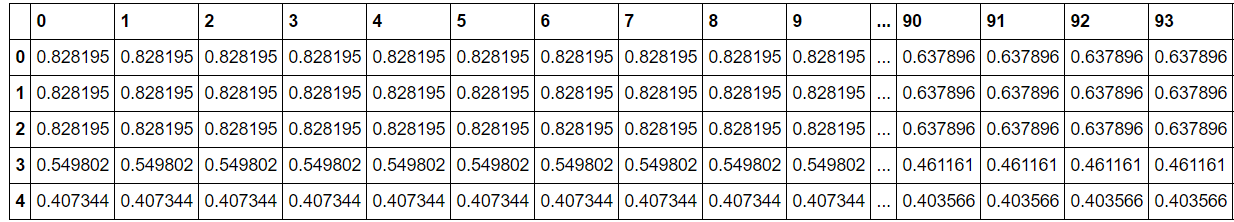
The no-arbitrage condition is not holding for all the points on the surface. As we can see, there are considerable dips and negative slopes in localized strike regions which could be potential arbitrage areas.

## D) Local Volatility

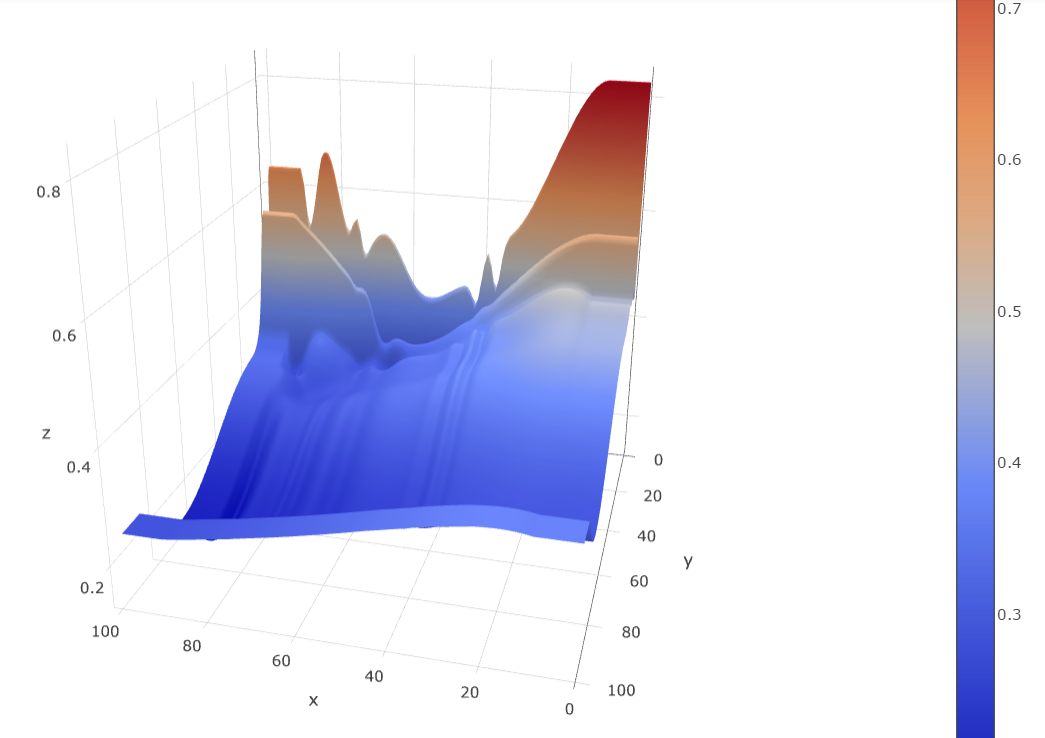




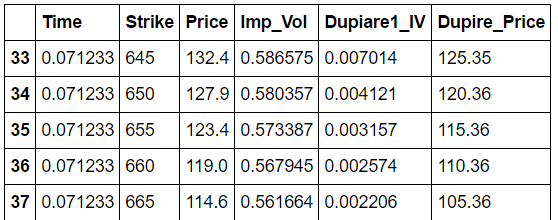
### Interpolated Dupire Local Volatility Grid



### Plot of Interpolated Dupire Volatility



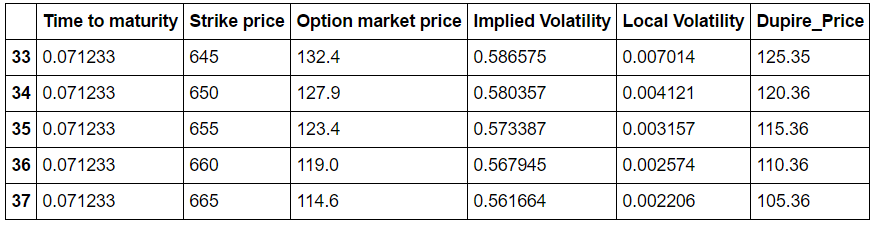
## E) Price of Option with Local Volatility



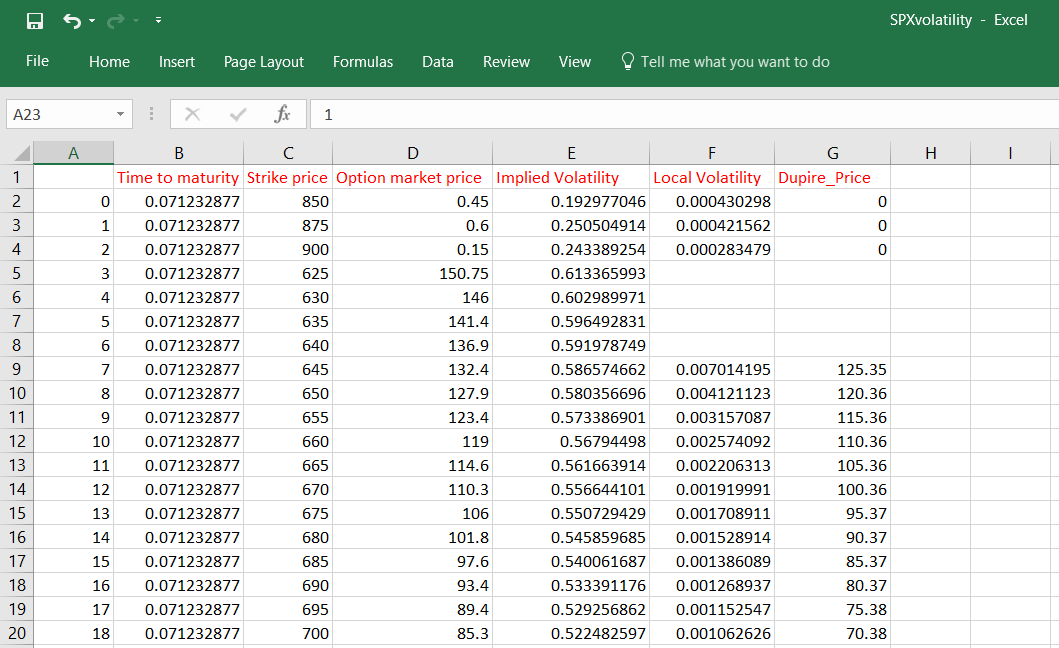
We can see that the Dupire price is less than the market price most of the times because we are taking the local volatility and local volatility is one that treats volatility as a function of both the current asset level and of time. As such, a local volatility model is a generalization of the Black-Scholes model, where the volatility is a constant. Because the only source of randomness is the stock price, local volatility models are easy to calibrate. Also, they lead to complete markets where hedging can be based only on the underlying asset. Since in local volatility models the volatility is a deterministic function of the random stock price, local volatility models are not very well used to price options whose values depend specifically on the random nature of volatility itself.

Therefore, Dupire option price is based on the underlying price volatility which is much lesser than the implied volatility and therefore the price calculated using Dupire volatility is less than the real price which is dependent on the implied volatility.

## F) Compiling all data into one table

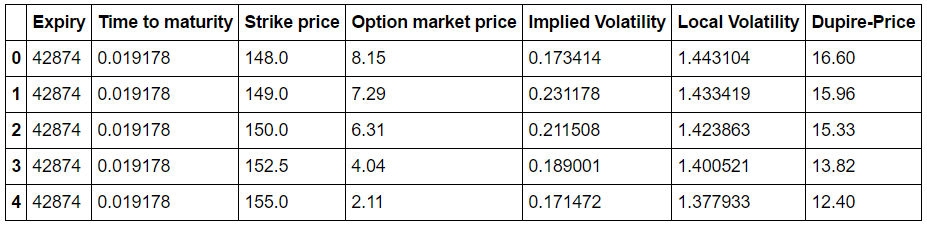


### "SPXvolatility.csv"



## G) Creating a custom function and Using External Data

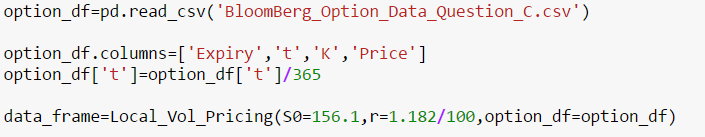




### Implementation Guide

File Name: **QuestionC\_PartG.py**

Code design: Python3.6



# Appendix:

# The code for these question (including the rmdb, jupyter notebook and HTML files) are in the corresponding folders in the submission file.